Equation Chapter 1 Section 1Lifeline design: calculation of the tensions

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Abstract. The aim of this paper is to evaluate the tensions of a system of horizontal lifeline by considering at the same time the elasticity in the lanyard, the lifeline and its anchorages, when these are flexible. The calculation is based on two energy principles: minimum total potential energy and energy conservation. This approach is easier to formulate, making possible the analytical and numerical solution of the three-component system. As to energy absorbers, four cases are discussed: energy absorber in the lanyard, in the lifeline, none, or both.

A method was developed for the numerical solution, using a spreadsheet software and its associated optimization algorithm, for minimization of the energy function. The method uses a readily available software, instead of a specific purpose one. It's possible to write the system parameters, the energy function and intermediate formulas directly on the cells, keeping the formulas simple. All results can also be computed by formulas in the cells. It's easy to try different values for the parameters and to incorporate new features to the model, by changing the energy function, and the variable cells.

In conclusion, it was shown that, by considering the elasticity of the anchorage, the maximum tension in the lifeline and on the anchorage can be significantly lower than the calculated without considering it, making it possible to design a lighter structure.

Keywords: lifeline; structural design; elasticity; energy; minimization; fall protection; construction; engineering

I. INTRODUCTION

Over 317 million accidents occur on the job annually worldwide and more than 2.3 million people are killed, according to ILO data[1]. The economic burden is estimated at 4 per cent of global Gross Domestic Product. These injuries can be avoided by adequate control measures.

Studies on fatal occupational accidents analyses in Brazilian State of Rio Grande do Sul[2,3], show that in 31.8% of fatal accidents, in all industries, falls were the immediate factor of mortality, and in construction industry this figure rises to 51.7% of the fatalities. Furthermore, it is shown that the absence of a proper design is a causal factor related to fall accidents.

The best way to avoid fall hazards is not to create them in first place. For example, the work can be planned to be done near the ground or in a place protected by walls. If this is not possible, a fall protection system must be prepared. Fall protection systems can be classified[4] in fall restriction systems and fall arrest systems. The fall restraint system, also known as travel restraint systems, provides protection by setting restraints on workers to prevent the fall from happening. They act before the fall happens. The fall arresting system is designed to function when a worker is in the process of falling. It provides protection by catching the worker during the fall and fully stopping the fall before the worker hits the lower level or an obstacle. In general, the fall arresting system requires more complex engineering and more specially designed components, in order to limit the impact upon the worker to avoid the injury caused by the system itself. Workers require more training to use fall arresting systems and rescue plans are necessary. Fall protection systems can furthermore be classified in active and passive. Active systems depend on some action being done by the user. Passive systems work independently of any user action. In general, collective protection is passive and individual protection, active. An example of passive fall restraint system are the guardrails at the edges of floors that are not closed by walls. A passive fall arrest system would be a safety net in order to catch a person that fell from an unprotected edge. An active fall restraint system example is to use a waist belt tied to a horizontal lifeline or rail by a lanyard which length does not allow the user to reach the edge of an inclined roof. An active fall arrest system example is to use a safety harness tied to a horizontal lifeline by a lanyard with shock absorber, in order to walk on a beam from where one can fall. If a fall restraint system is feasible and offers complete protection, it is preferable to a fall arrest one. Likewise, a passive system is preferable to an active.

One of the components that an active fall arrest or fall restraint system may have are horizontal lifelines. A horizontal lifeline (HLL) is a line that is stretched between two extremes of travel in a workplace, to provide a continuous anchor for the attachment of fall-arrest equipment. They are needed for several reasons, such as providing horizontal traveling and avoiding swing fall hazard.

The basic quantities in HLL calculations are the tension in the lanyard and the tension in the line. The first one is usually determined like in any active fall arrest system. When the lanyard is equipped with an energy absorber, the tension in the lanyard is the absorber peak force. Otherwise, the tension is given by the so called Standard Equation for Impact Force [5]. The tension in the line, however, in the case of flexible horizontal lifelines, is different from the tension in the lanyard, because the latter is transversal to the line[6,7].

Research on HLL, both theoretical and testing, has been made, since the seventies[8,9,10,11], but still there are open questions. Horizontal lifeline design, specially for fall arrest, can be a difficult problem. The value of tension in the lifeline is usually high, specially for taut low-sag lines, which affects not only the dimensioning of the cable, but also of the anchorages. It's a nonlinear dynamical problem. Many parameters are involved where a small difference can largely modify the results. The parameters that will be dealt with in this paper are the elasticity in the lanyard, the lifeline and the anchorage. The elasticity of the lanyard increases arresting time and distance, thus reducing the arresting tension in the lanyard, and by consequence also in the line. The elasticity of the lifeline reduces the value of the tension in the line during fall arrest not only by increasing arresting time but mainly by augmenting dynamical sag. But in some cases the anchorages are also flexible. Then the tension can be further reduced by taking anchorage elasticity in consideration.

The flexibility of anchorages had already been studied by McIntyre, Arteau, Lan and Corbeil[12,13,14,15,16]who, in tests using posts as the end-anchoring means, demonstrated that the degree of rigidity (or flexibility) of the end-anchor arrangement can affect performance. The weight of the anchor posts (in the order of 50 kg) created difficulties in handling and installation, hence the need to create a light and safe anchor pole capable weight to withstand the fall a worker. To combine lightness and resistance, the post should undergo plastic deformation to absorb the energy of the fall. Paureau and Jacqmin[17], by their turn, studied lifelines anchored in vertical posts, equipped with elastic devices, as springs, between the lifeline and the anchorage, in order to augment the lifeline sag during the fall arrest.

The present paper is a theoretical study on horizontal lifelines fixed in anchorages that have some degree of flexibility, for example a vertical post that suffers a finite displacement when pulled by the lifeline cable during the fall arrest. The aim is to evaluate the tensions by considering at the same time the elasticity in the lanyard, in the lifeline and in the anchorage.

II. FORMULATION

We'll consider a system (fig. 1) with three components:

1. A vertical lanyard connected to the middle of
2. a horizontal lifeline, fixed in both ends to
3. the anchorages.

We assume each component is elastic and weightless. The lanyard has an initial length l_{1_0}. When submitted to a tension T_1, it stretches to a final length l_1. Its variation is $\Delta l_1 = l_1 - l_{1_0}$. We assume it obeys Hooke's law, $T_1 = k_1 \Delta l_1$, where k_1 is the spring constant. The same with the lifeline: $T_2 = k_2 \Delta l_2$, where T_2 is the longitudinal tension, along the cable. The anchorage is of a kind that will yield some lateral displacement when submitted to the horizontal tension from the lifeline. Like, for example, a metal pole with the bottom end fixed and the top end free. When an unitary horizontal force is applied, it yields a displacement δ. As this displacement occurs at both sides, the length of the span l_{3_0} diminishes twice this quantity: $\Delta l_3 = l_{3_0} - 2\delta$, for a unitary tension.

Then, we can consider the span as a spring under compression: $T_3 = k_3 \Delta l_3$, where

$$\Delta l_3 = l_{3_0} - l_3 \text{ and } k_3 = \frac{1}{2\delta}$$

We want to calculate the lengths variations and the tensions in each component, at their maximum, in the moment when the falling mass momentarily stops before bouncing upwards. To do this, we will apply in the end of the lanyard a force F, equal to the tension in the lanyard at this moment. Then we will resolve the static problem rather than the dynamic one.

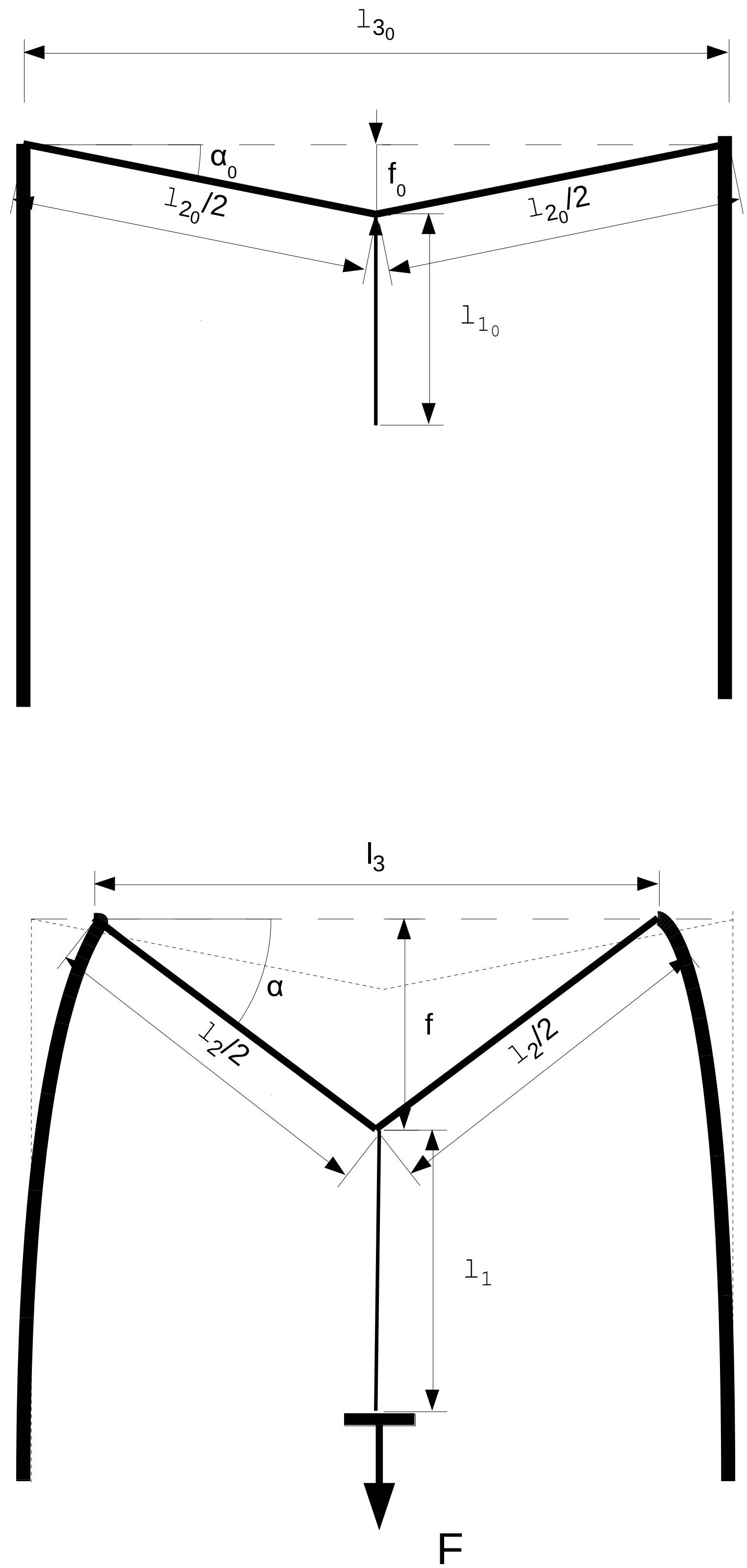

FIG. 1: The system is compound by a lanyard(1), a horizontal lifeline (2) and the span between anchorages(3). Above, the system unloaded, and below, the system subject to a force F.

So, the parameters (inputs) are:
the elastic constants

$$k_1, k_2, k_3 \tag{1}$$

, the initial (unloaded) lengths

$$l_{1_0}, l_{2_0}, l_{3_0} \tag{2}$$

, and the applied force

$$F \tag{3}$$

The variables (unknowns) are the length variations

$$\Delta l_1, \Delta l_2, \Delta l_3 \tag{4}$$

which are to be determined. From them, we obtain the final (loaded) lengths

$$\begin{aligned} l_1 &= l_{1_0} + \Delta l_1 \\ l_2 &= l_{2_0} + \Delta l_2 \\ l_3 &= l_{3_0} - \Delta l_3 \end{aligned} \tag{5}$$

and the final tensions

$$T_i = k_i \Delta l_i \tag{6}$$

We apply an external force F, which later we'll make equal to the impact force, and try to find the distribution of lengths and tensions when the system is in equilibrium with this force.

For this purpose, we use the principle of minimum total potential energy[18], which can be stated as:

Among all the possible displacements consistent with the reactions, the correct state of displacement is that which minimizes the total potential energy.

This method is equivalent to the usual equilibrium conditions of null resultant force and moment, but sometimes leads to simpler calculations and permits a deeper physical insight.

The elastic potential energy is

$$U_e = \frac{k_1}{2}(\Delta l_1)^2 + \frac{k_2}{2}(\Delta l_2)^2 + \frac{k_3}{2}(\Delta l_3)^2. \tag{7}$$

The potential energy function of the external loads is:

$$U_F = -F(f + l_1), \tag{8}$$

where f is the sag:

$$f = \frac{1}{2}\sqrt{l_2^2 - l_3^2}, \tag{9}$$

provided $l_2 \geq l_3$, which is always true, since the lifeline is attached to the anchorage.

The total potential energy is

$$U = U_e + U_F. \tag{10}$$

When the system is in equilibrium, this function is a minimum.

III. SOLVING

A. Solving analytically

If the total potential energy function U is a minimum, than $dU = 0$, which means that every partial derivative must vanish:

$$\frac{\partial U}{\partial l_1} = 0 \Rightarrow k_1(l_1 - l_{1_0}) - F = 0 \tag{11}$$

$$\frac{\partial U}{\partial l_2} = 0 \Rightarrow k_2(l_2 - l_{2_0}) - \frac{F}{2}\frac{l_2}{\sqrt{l_2^2 - l_3^2}} = 0 \tag{12}$$

$$\frac{\partial U}{\partial l_3} = 0 \Rightarrow k_3(l_{3_0} - l_3) - \frac{F}{2}\frac{l_3}{\sqrt{l_2^2 - l_3^2}} = 0 \tag{13}$$

The force equations are obtained without dealing with vector components or trigonometry. It all comes out of the energy function.

The first equation is already decoupled, and the other two form a set. From this set, the values of l_2 and l_3 can be obtained, at least numerically, and from them; T_2 and T_3, and it's done.

The following trigonometric relations can be drawn from lower figure 1:

$$\cos\alpha = l_3 / l_2 \tag{14}$$

$$\sin\alpha = 2f / l_2 \tag{15}$$

and also, if $l_{2_0} \geq l_{3_0}$,

$$\cos\alpha_0 = l_{3_0} / l_{2_0}$$

Substituting (6), (9), (14) and (15) in (11) to (13), we get

$$T_1 = F \tag{16}$$

$$T_2 = \frac{F}{2\sin\alpha} \tag{17}$$

$$T_3 = \frac{F}{2\tan\alpha} \tag{18}$$

and from (17) and (18),

$$T_3 / T_2 = l_3 / l_2 = \cos\alpha \tag{19}$$

The force equations are obtained without dealing with vector components or trigonometry. The energy function has all the information about the system.

Substituting eqs. (11) to (13), (14) and (15) in (7), we get:

$$U_e = \frac{F^2}{2}\left\{\frac{1}{k_1} + \frac{1}{4k_2\sin^2\alpha} + \frac{1}{4k_3\tan^2\alpha}\right\}$$

Whence, the equivalent stiffness k_e of the three springs system can be defined by

$$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{4k_2\sin^2\alpha} + \frac{1}{4k_3\tan^2\alpha}$$

The stiffness increases with the angle,

from $\lim_{\alpha\to 0} k_e = 0$ to $\lim_{\alpha\to\pi/2} k_e = \dfrac{1}{\dfrac{1}{k_1} + \dfrac{1}{4k_2}}$.

From eqs. (12) and (13), doing some algebra, adding both equations and diminishing one from the other, we get:

$$f\left(\frac{l_{3_0}}{l_3}-\frac{l_{2_0}}{l_2}\right)=\left(\frac{1}{k_2}+\frac{1}{k_3}\right)\frac{F}{4} \tag{20}$$

$$k_2\frac{l_{2_0}}{l_2}+k_3\frac{l_{3_0}}{l_3}=k_2+k_3 \tag{21}$$

Substituting (14) and (15) into eqs. (20) and (21):

$$tan(\alpha)-\frac{sin(\alpha)}{l_{3_0}/l_{2_0}}=\left(\frac{1}{k_2}+\frac{1}{k_3}\right)\frac{F}{2l_{3_0}}, \tag{22}$$

$$k_3tan(\alpha)+k_2\frac{sin(\alpha)}{l_{3_0}/l_{2_0}}=(k_2+k_3)\frac{2f}{l_{3_0}}, \tag{23}$$

Eq. (22) corresponds to the equation derived by Paureau and Jacqmin[19], where it is called the law of lifeline behavior.

If $l_{3_0}\le l_{2_0}$,we can define $cos(\alpha_0)=l_{3_0}/l_{2_0}$, where α_0 corresponds to the initial angle.

$$tan(\alpha)-\frac{sin(\alpha)}{cos(\alpha_0)}=\left(\frac{1}{k_2}+\frac{1}{k_3}\right)\frac{F}{2l_{3_0}}, \tag{24}$$

$$k_3tan(\alpha)+k_2\frac{sin(\alpha)}{cos(\alpha_0)}=(k_2+k_3)\frac{2f}{l_{3_0}}, \tag{25}$$

The angle α is obtained from (24) (or from (22)). From (25) (or from (23)), the sag f is obtained. From (14), (15), (16), (17) and (18), we get l_2, l_3, T_1, T_2, T_3.

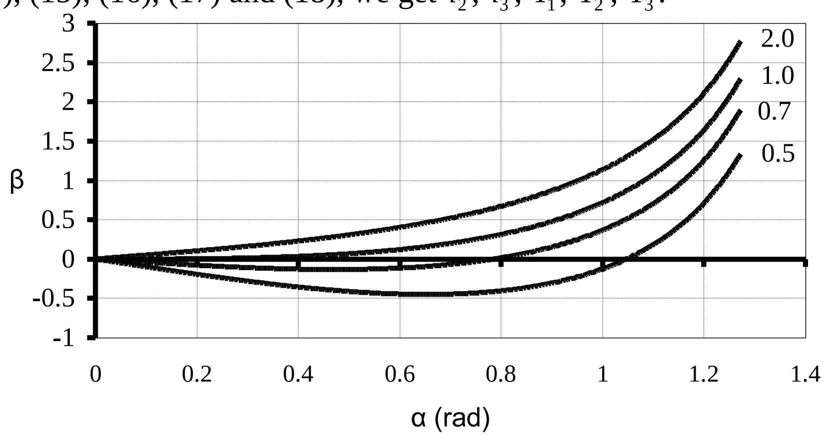

FIG. 2: Graph of the function $\beta=tan(\alpha)-\frac{sin(\alpha)}{l_{3_0}/l_{2_0}}$, for several values of l_{3_0}/l_{2_0}

In Fig. 1 we plot the function $\beta=tan(\alpha)-\frac{sin(\alpha)}{l_{3_0}/l_{2_0}}$, for several values of l_{3_0}/l_{2_0}. The final angle α that results from a applied force F can be determined as the value in the abscissa corresponding to a value $\beta = \left(\frac{1}{k_2} + \frac{1}{k_3} \right) \frac{F}{2 l_{3_0}}$ in the ordinate. For $l_{3_0} / l_{2_0} \le 1$, the curves cut the abscissa in the value of the initial angle α_0.

If $l_{2_0} < l_{3_0}$, there is tension in the lifeline and in the anchorage when $F = 0$. Let's calculate the tension in this situation. For $l_{3_0} / l_{2_0} > 1$, and $F = 0$, we see in the fig. 2 that $\alpha = 0$, so the line is straight, and we have:

$$l_2 = l_3 = l \tag{26}$$

From (19):

$$T_2 = T_3 = T \tag{27}$$

From (26), (27), (5) and (6):

$$T = \frac{l_{3_0} - l_{2_0}}{\frac{1}{k_2} + \frac{1}{k_3}} \tag{28}$$

$$l = \frac{k_2 l_{2_0} + k_3 l_{3_0}}{k_2 + k_3} \tag{29}$$

And the elastic potential energy is:

$$U_{e_0} = \frac{T^2}{2} \left(\frac{1}{k_2} + \frac{1}{k_3} \right) \tag{30}$$

Having solved the problem for a given applied force, this one is made equal to the maximum impact force that occurs during the fall arrest. Now, we have to consider the presence of energy absorbers in the system. We assume an ideal energy absorber (fig. 3), in which, once the activation tension is reached, the energy absorber begins to yield and the tension T_a is kept constant until the absorber extension limit $\Delta l_{a,\max}$ is reached.

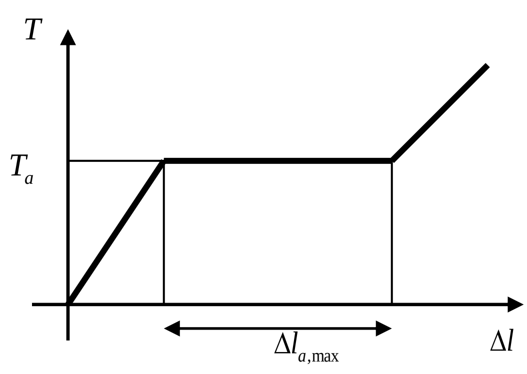

FIG. 3: Ideal energy absorber of constant tension T_a and maximum extension $\Delta l_{a,\max}$.

There are four cases: energy absorber in the lanyard, in the lifeline, none, or both.

1. Energy absorber in the lanyard

We identify the absorber yield tension T_a with the applied force F used in the calculation above, $F = T_a$, then obtaining the results Δl_i, l_i, and T_i as above.

Then, we find the absorber extension Δl_a , using the energy balance:

$$\Delta l_a = \frac{mg\left(y_0 + f + l_1\right) - \Delta U_e}{T_a - mg}, \tag{31}$$

where $$\Delta U_e = \left[\frac{k_1}{2}(\Delta l_1)^2 + \frac{k_2}{2}(\Delta l_2)^2 + \frac{k_3}{2}(\Delta l_3)^2 - U_{e_0}\right], \tag{32}$$

and U_{e_0} is zero, if $l_{2_0} \geq l_{3_0}$, or U_{e_0} is from (30), if $l_{2_0} < l_{3_0}$.

If $\Delta l_a < 0$, then the lanyard energy absorber wasn't activated, and we proceed to the calculation as in the case with no absorber.

If $\Delta l_a > \Delta l_{a,\max}$, this means that the lanyard energy absorber was fully torn and yet the mass did not stop. Then, we calculate as the case with no absorber, but diminishing $T_a \Delta l_a / mg$ from the free fall distance, in order to discount the energy dissipated by the absorber, and adjusting l_{1_0} and k_1 , as appropriate.

2. No energy absorber

Let y_0 be the height of the point of connection of the lanyard to the worker, before the fall, above the horizontal line that passes by the lifeline anchorages. The total fall distance, until complete stop, is $y_0 + f + l_1$. During the fall, the gravitational potential energy converts into kinetic energy, which converts to elastic potential energy during fall arresting. In the moment of stopping, we can equate the gravitational potential energy variation ΔU_g and elastic potential energy variation ΔU_e :

$$mg\left(y_0 + f + l_1\right) - \Delta U_e = 0 \tag{33}$$

At the same time, we have the maximum fall arrest force. So, we have to find the value of F for which condition (33) is fulfilled.

3. Energy absorber in the lifeline

We set tension T_2 in the lifeline equal to the tension T_a of the absorber.

$$T_2 = T_a \tag{34}$$

From (19): $$T_3 = T_a \frac{l_3}{l_2} \tag{35}$$

From (5) and (6): $$l_2 = l_{2_0} + \frac{T_a}{k_2} + \Delta l_a \tag{36}$$

$$l_3 = l_{3_0} - \frac{T_3}{k_3} \tag{37}$$

From (34) to (37):

$$T_3 = \frac{l_{3_0}}{\dfrac{l_{2_0}}{T_a} + \dfrac{1}{k_2} + \dfrac{1}{k_3} + \dfrac{\Delta l_a}{T_a}} \tag{38}$$

$$l_3 = \frac{T_3}{T_a} l_2 \tag{39}$$

$$T_1 = \frac{4f}{l_2} T_2 \tag{40}$$

$$l_1 = l_{1_0} + \frac{T_1}{k_1} \tag{41}$$

$$f = \frac{1}{2}\sqrt{l_2^2 - l_3^2} \tag{42}$$

The absorber continues to yield, extending Δl_a, thus increasing lifeline length and angle α. During this process, tension T_2 is held constant, but tension T_1 in the lanyard is increasing, because the angle is increasing. Eventually, the mass stops. We make the energy balance:

$$\Delta l_a T_a = mg\left(y_0 + f + l_1\right) - \Delta U_e \tag{43}$$

where ΔU_e is the same as in eq. (32).

The right member of eq. (43) has terms that depend on equations (36) to (42), which depend on the absorber extension Δl_a, so, in order to find this quantity, all equations must be solved together.

If $\Delta l_a < 0$, then the lifeline energy absorber wasn't activated, and we proceed to the calculation as in the case with no absorber.

If $\Delta l_a > \Delta l_{a,\max}$, this means that the absorber in the lifeline was fully depleted and yet the mass did not stop. Then, we calculate as the case with no absorber, but adjusting the free fall distance, diminishing the energy dissipated by the absorber, $T_a \Delta l_a / mg$, and adjusting l_{2_0} and k_2, as well.

4. Energy absorber in both lanyard and lifeline

The angle α_2 for which both absorbers will be yielding is obtained from:

$$sin(\alpha_2) = \frac{T_{ly}}{2T_{ll}} \tag{44}$$

where T_{ly} is the lanyard absorber yield tension and T_{ll} is the lifeline absorber yield tension.

If the initial angle $\alpha_0 \geq \alpha_2$, then lanyard absorber fires first, the tension will be kept constant in both the lanyard and the lifeline, so the lifeline absorber will not fire. We proceed the calculation as in case 1.

If $\alpha_0 < \alpha_2$, then the lifeline absorber fires first. The lanyard tension will continue to increase as the angle increases. When the angle α_2 is reached, the lanyard absorber's firing tension is reached, and it will begin to yield. In this moment, the lifeline absorber will stop yielding.

We can evaluate the variables at the moment when $\alpha = \alpha_2$. From (5), (6), (14), (19),:

$$cos(\alpha_2) = \frac{l_3}{l_2} \tag{45}$$

$$cos(\alpha_2) = \frac{T_3}{T_{ll}} \tag{46}$$

$$l_3 = \frac{T_3}{T_{ll}} l_2 \quad (47)$$

$$l_3 = l_{3_0} - \frac{T_{ll}\cos(\alpha_2)}{k_3} \quad (48)$$

$$l_2 = l_{2_0} + \frac{T_{ll}}{k_2} + \Delta l_{ll} \quad (49)$$

The lifeline absorber extension Δl_{ll} when the lanyard yields is given by:

$$\Delta l_{ll} = \left(\frac{l_{3_0}/l_{2_0}}{\cos(\alpha_2)} - 1 \right) l_{2_0} - \left(\frac{1}{k_2} + \frac{1}{k_3} \right) T_{ll} \quad (50)$$

If, from (45), we find that $\Delta l_{ll} < 0$, then the lifeline absorber doesn't fire, and the calculation is as in the section where only the lanyard has absorber. If $\Delta l_{ll} > \Delta l_{ll,\max}$, then the lifeline absorber will be depleted before the lanyard's has been activated.

The lanyard absorber extension Δl_{ly} is given by the energy balance:

$$\Delta l_{ly} = \frac{mg\left(y_0 + f + l_1\right) - \Delta U_e - T_{ll}\Delta l_{ll}}{T_{ly} - mg}, \quad (51)$$

where f is given by eq.(42), and

$$l_1 = l_{1_0} + \frac{T_{ly}}{k_1} \quad (52)$$

B. Solving numerically with minimization algorithm in worksheet software.

We can solve eq. (22) numerically, finding the angle α for a given applied force, and from it, all other quantities. But we can also go back to eq. (10), and minimize the potential energy function, directly obtaining the displacements. Any algorithm or software suitable for multi-variable non-linear optimization can be used. In this paper, we have used the Microsoft Excel Solver.

1. Energy absorber in the lanyard

In a worksheet, we write in the cells (fig. 4) the values of the parameters (1) to (3), where, in the case of lanyard with energy absorber, the applied force F is set to the maximum arrest force of the absorber. For the displacements Δl_i (4), which are the variables to be determined, we write initial guess values, let's say, 0. In other cells, we write the formulas (5) to (10).

In the Solver parameters (fig. 5), we set the Objective cell to the total potential energy cell (10); choose "Min"; and set the Variable Cells, to the Δl_i (4). In Options, we uncheck “Assume linear model”, and we do check “Assume non-negative”. In a glimpse, we get the results (fig. 6), in the cells (4), the values of the displacements Δl_i; in the cells (5), the final lengths l_i; and, in the cells (6), the tensions T_i, in the lanyard, in the lifeline cable, and in the span, which is the horizontal force transmitted by the lifeline to the anchorage. We also calculate the absorber extension Δl_a, by using energy conservation equation (31).

Arquivo Editar Exibir Inserir Formatar Ferramentas Dados Janela Ajuda

	A	B	C	D	E	F	G	H	I
1		lanyard	lifeline	span	sag	sum	units		
2	ref.	1	2	3	f			input data	value
3	F	6000					N	variables	value
4	k_i	30000	1200000	80000			N/m	to minimize	formula
5	l_{i0}	1.2	6.23	6.2	=0.5*(C5^2-D5^2)^0.5		m	results	formula
6	Δl	0	0	0			m		
7	l_i	=B5+B6	=C5+C6	=D5-D6	=0.5*(C7^2-D7^2)^0.5		m	initial sag f_0 %	=E5/D5
8	U_e	=B4/2*B6^2	=C4/2*C6^2	=D4/2*D6^2		=SOMA(B8:E8)	J	final sag f %	=E7/C5
9	U_f	=-B7*$B3			=-E7*$B3	=SOMA(B9;E9)	J	cable elongation e_2 %	=C6/C5
10	U					=SOMA(F8:F9)	J	span shortening e_3 %	=D6/D5
11	T_i	=B4*B6	=C4*C6	=D4*D6			N		
12									
13	mass	gravity	initial position	free fall distance	fall factor	grav. energy		absorber extension	
14	m (kg)	g (m/s²)	y_0 (m)	$h= y_0 + l_{10} + f_0$	$r = h / l_{10}$	$U_g=mg(y_0 + l_1 + f)$ (J)	U_g-U_e	Δl_a (m)	$\Delta l_{a, max}$
15	100	9.81	=0.6-E5	=C15+B5+E5	=D15/B5	=A15*B15*(C15+B7+E7)	=F15-F8	=G15/(B3-B15*C15)	0.45
16									

FIG. 4: Write the values and the formulas in the worksheet.

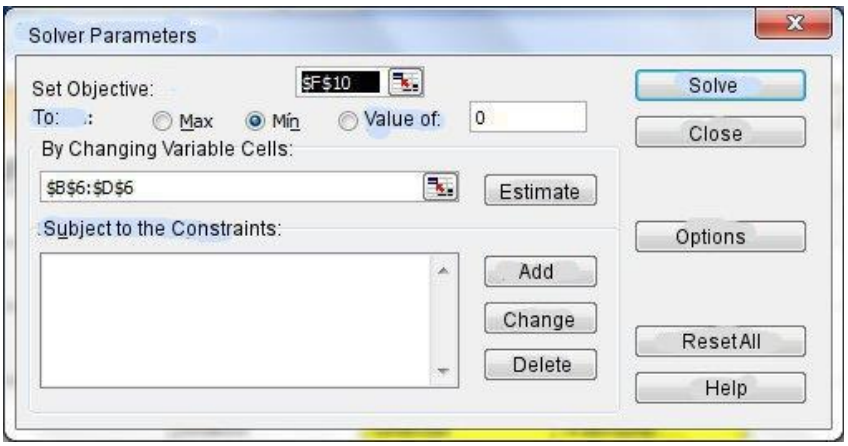

Fig. 5: Set Solver parameters and execute.

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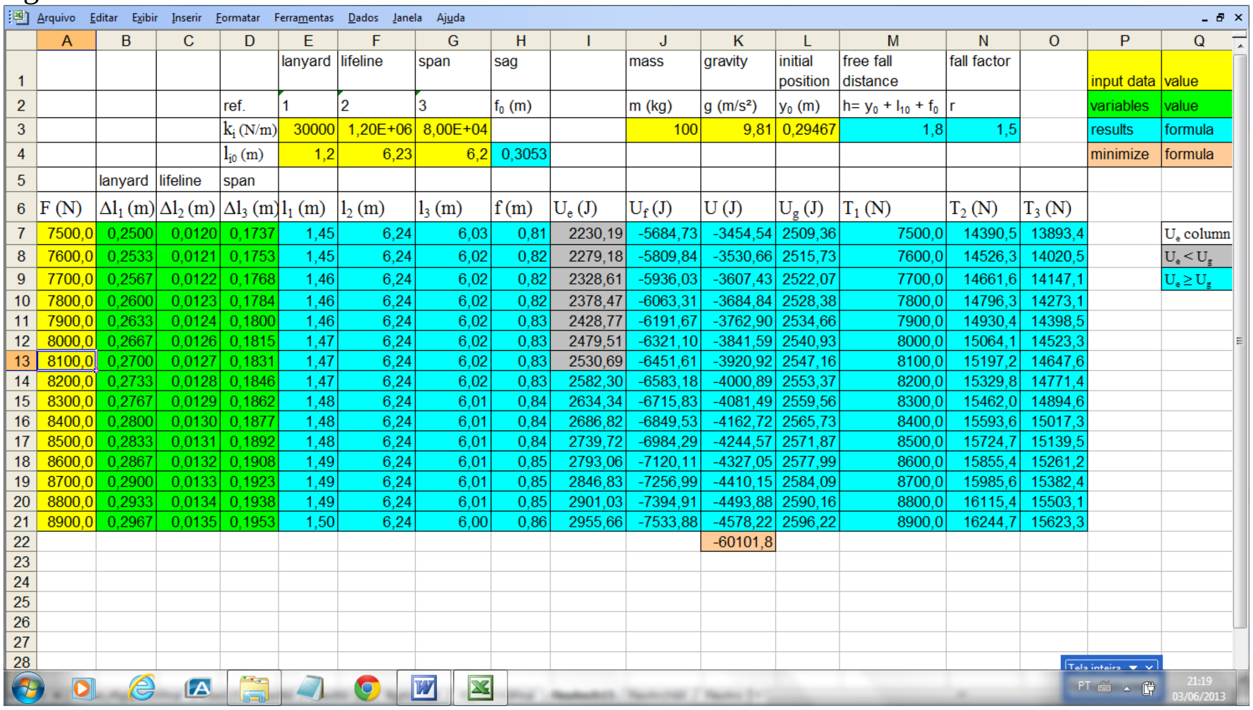
	A	B	C	D	E	F	G	H	I
1		lanyard	lifeline	span	sag	sum	units		
2	ref.	1	2	3	f			input data	value
3	F	6000					N	variables	value
4	k_i	30000	1200000	80000			N/m	to minimize	formula
5	l_{i0}	1.2000	6.2300	6.2000	0.3053		m	results	formula
6	Δl	0.2000	0.0102	0.1488			m		
7	l_i	1.4000	6.2402	6.0512	0.7622		m	initial sag f_0 %	4.9%
8	U_e	600	63	886		1549	J	final sag f %	12.2%
9	U_f	-8400			-4573	-12973	J	cable elongation e_2 %	0.16%
10	U					-11424	J	span shortening e_3 %	2.40%
11	T_i	6000	12280	11908			N		
12									
13	mass	gravity	initial position	free fall distance	fall factor	grav. energy	absorp tion	absorber extension	
14	m (kg)	g (m/s²)	y_0 (m)	$h= y_0 + l_{10} + f_0$	$r = h / l_{10}$	Ug (J)	U_g-U_e	Δl_a (m)	$\Delta l_{a, max}$ (m)
15	100	9.81	0.2947	1.8000	1.5	2410	861	0.1716	0.45

FIG. 6: Results.

2. No energy absorber

For the case without any energy absorber, we don't know *a priori* the impact force F in the lanyard. So, we try every value, finding the corresponding displacements and forces, and checking the balance between gravitational and elastic energy, by eq. (33), as discussed in case 2 of section A above, in order to determine which is the value of F that is consistent with the free fall distance. As before, we write in the cells the values of parameters (1) k_1, k_2, k_3, (2) $l_{1_0}, l_{2_0}, l_{3_0}$, and, for evaluating (33), also the mass m, the gravitational acceleration g, and the initial position of the worker y_0, measured above the line between anchorages. Then, we write, several values of (3) F, one in each line, as well as the corresponding guess values of the variables $\Delta l_1, \Delta l_2, \Delta l_3$, and the formulas for the quantities (5) to (10), and also the gravitational potential energy U_g. We sum in a cell all the cells of the total potential energy (10) U of each line. Now, we set in the Solver parameters window this cell as the Objective cell to be minimized, and all of the cells of the displacements $\Delta l_1, \Delta l_2, \Delta l_3$ of all lines as the variable cells. When we execute the target cell is minimized, all the cells of the total potential energy U (10) of each line will be minimized simultaneously, and the displacements in each line will become consistent with the corresponding applied force. Then, we compare the elastic potential energy with the gravitational potential energy in each line. When they are equal, the value of the force F of that line is the impact force consistent with the free fall distance, and all other quantities of that line will be the right ones.



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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
1					lanyard	lifeline	span	sag		mass	gravity	initial position	free fall distance	fall factor		input data	value
2				ref.	1	2	3	f_0 (m)		m (kg)	g (m/s²)	y_0 (m)	$h = y_0 + l_{10} + f_0$	r		variables	value
3				k_i (N/m)	30000	1,20E+06	8,00E+04			100	9,81	0,29467	1,8	1,5		results	formula
4				l_{i0} (m)	1,2	6,23	6,2	0,3053								minimize	formula
5		lanyard	lifeline	span													
6	F (N)	Δl_1 (m)	Δl_2 (m)	Δl_3 (m)	l_1 (m)	l_2 (m)	l_3 (m)	f (m)	U_e (J)	U_f (J)	U (J)	U_g (J)	T_1 (N)	T_2 (N)	T_3 (N)		
7	7500,0	0,2500	0,0120	0,1737	1,45	6,24	6,03	0,81	2230,19	-5684,73	-3454,54	2509,36	7500,0	14390,5	13893,4		U_e column
8	7600,0	0,2533	0,0121	0,1753	1,45	6,24	6,02	0,82	2279,18	-5809,84	-3530,66	2515,73	7600,0	14526,3	14020,5		$U_e < U_g$
9	7700,0	0,2567	0,0122	0,1768	1,46	6,24	6,02	0,82	2328,61	-5936,03	-3607,43	2522,07	7700,0	14661,6	14147,1		$U_e \geq U_g$
10	7800,0	0,2600	0,0123	0,1784	1,46	6,24	6,02	0,82	2378,47	-6063,31	-3684,84	2528,38	7800,0	14796,3	14273,1		
11	7900,0	0,2633	0,0124	0,1800	1,46	6,24	6,02	0,83	2428,77	-6191,67	-3762,90	2534,66	7900,0	14930,4	14398,5		
12	8000,0	0,2667	0,0126	0,1815	1,47	6,24	6,02	0,83	2479,51	-6321,10	-3841,59	2540,93	8000,0	15064,1	14523,3		
13	8100,0	0,2700	0,0127	0,1831	1,47	6,24	6,02	0,83	2530,69	-6451,61	-3920,92	2547,16	8100,0	15197,2	14647,6		
14	8200,0	0,2733	0,0128	0,1846	1,47	6,24	6,02	0,83	2582,30	-6583,18	-4000,89	2553,37	8200,0	15329,8	14771,4		
15	8300,0	0,2767	0,0129	0,1862	1,48	6,24	6,01	0,84	2634,34	-6715,83	-4081,49	2559,56	8300,0	15462,0	14894,6		
16	8400,0	0,2800	0,0130	0,1877	1,48	6,24	6,01	0,84	2686,82	-6849,53	-4162,72	2565,73	8400,0	15593,6	15017,3		
17	8500,0	0,2833	0,0131	0,1892	1,48	6,24	6,01	0,84	2739,72	-6984,29	-4244,57	2571,87	8500,0	15724,7	15139,5		
18	8600,0	0,2867	0,0132	0,1908	1,49	6,24	6,01	0,85	2793,06	-7120,11	-4327,05	2577,99	8600,0	15855,4	15261,2		
19	8700,0	0,2900	0,0133	0,1923	1,49	6,24	6,01	0,85	2846,83	-7256,99	-4410,15	2584,09	8700,0	15985,6	15382,4		
20	8800,0	0,2933	0,0134	0,1938	1,49	6,24	6,01	0,85	2901,03	-7394,91	-4493,88	2590,16	8800,0	16115,4	15503,1		
21	8900,0	0,2967	0,0135	0,1953	1,50	6,24	6,00	0,86	2955,66	-7533,88	-4578,22	2596,22	8900,0	16244,7	15623,3		
22											-60101,8						

FIG. 7: Results for the case of none energy absorber. Each line corresponds to a guess value of impact force. The one that will really occur is the line where $U_e = U_g$, in this example, between 8100 and 8200 N.

IV. DISCUSSION

A. Comparing results

1. No energy absorber

We've ran the case of no energy absorber, as described in B.2 above, with the following parameters:

TABLE 1: Parameters used in the example calculations.

	lanyard	lifeline	span	sag	rel. sag	mass	free fall dist.	fall factor
ref.	1	2	3	f_0	f_0%	m (kg)	$h=y_0+l_{10}+f_0$	$r=h/l_{10}$
k_i	30000	1.20E+06	8.00E+04	0.3053	4.92%	100	1.8	1.5
l_{i0}	1.2	6.23	6.2					

For comparison, we've ran three cases. In the first case, the lengths of lanyard, the lifeline and the span could vary. In the second case, we didn't permit the span length to vary, simulating a rigid anchorage condition. This can be done by setting the Solver Variable cells only to $\Delta l_1, \Delta l_2$ cells, and setting $\Delta l_3 = 0$. In the third case, we've permitted only the lanyard Δl_1 to vary, keeping fixed the other two lengths. Below, we show only the right result line for each case (the one which satisfies energy conservation condition).

TABLE 2: Results for the case of none energy absorber.

	cases	F (N)	Δl_1	Δl_2	Δl_3	l_1	l_2	l_3	f	f%	e_2%	e_3%	T_1	T_2	T_3
i	1, 2, 3 vary	8133	0.27	0.01	0.18	1.47	6.24	6.02	0.83	13.43%	0.20%	2.96%	8133	15241	14688
ii	1, 2 vary; 3 fixed	10016	0.33	0.03	0.00	1.53	6.26	6.20	0.43	6.98%	0.48%	0.00%	10016	36207	0
iii	1 varies; 2, 3 fixed	11320	0.38	0.00	0.00	1.58	6.23	6.20	0.31	4.92%	0.00%	0.00%	11320	0	0

We see that:

a) The impact force calculated by the method in case (iii), 11320 N, reproduces the value obtained from the Standard Equation of the Impact Force for a mass and a lanyard in a fixed point:

$$F = mg\left(1+\sqrt{1+\frac{2kr}{mg}}\right), \tag{53}$$

where k, the rope modulus in N, is given by the product of stiffness constant of the lanyard k_1, 30000 N/m, by its length l_1, 1.20 m and r is the fall factor $\left(r = h/l_{1_0} = 1.8m/1.2m = 1.5\right)$.

b) Case (ii) reproduces the result for the tension T_2 in the lifeline that would be obtained if, departing from the impact force of $T_1 =$ 10016 N, we would apply iteratively the equations

$$T_2 = \frac{Fl_2}{4f} \tag{54}$$

and

$$l_2 = l_{2_0} + \frac{T_2}{k_2} \tag{55}$$

However, if we would depart from the value obtained from eq. (53), 11320N, and apply iteratively (54) and (55), we would arrive on a 10% higher value for T_2.

c) The impact force in the lanyard, T_1, in case (i) is 19% less than case (ii), which is 12% less than (iii), because the energy is partially absorbed by the elastic deformations of the lifeline and the anchorage.

d) The tension in the lifeline, considering anchorage flexibility (case i) was 58% less than without considering it (case ii). This is so, not only because part of the energy is absorbed by the anchorage, which reduced the impact force in the lanyard, but also because of the greater sag (13.48% of the span, in case i, compared to 6.98%, in case ii). This is because the flexibility of the anchorage is bigger than that of the cable. In case (i), the cable stretched only 0.2%, while the span contracted 2.96%).

2. Energy absorber in the lanyard

With the same parameters as the precedent case, and an absorber tension T_a of 6000 N, we've set the applied force F to that tension, again running three cases. In the first case, the lanyard, the lifeline and the span lengths could vary. In the second case, we didn't permit the span length to vary. In the third case, we've permitted only the lanyard Δl_1 to vary, keeping fixed the other two lengths. The following results were produced.

TABLE 3: Results for the case of energy absorber in the lanyard.

	cases	Δl_1	Δl_2	Δl_3	l_1	l_2	l_3	f	f%	e_2%	e_3%	T_1	T_2	T_3	Δl_a
i	1, 2, 3 vary	0.20	0.010	0.15	1.40	6.24	6.05	0.76	12.24%	0.16%	2.40%	6000	12280	11908	0.17
ii	1, 2 vary; 3 fixed	0.20	0.020		1.40	6.25	6.20	0.39	6.32%	0.32%	0.00%	6000	23803	0	0.24
iii	1 vary; 2, 3 fixed	0.20			1.40	6.23	6.20	0.31	4.90%	0.00%	0.00%	6000	0	0	0.27

We see that:

a) Case (ii) reproduces the result for the tension T_2 in the lifeline that would be obtained if, departing from an impact force of 6000 N, we would apply iteratively the equations (54) and (55).

b) The tension in the lifeline, considering anchorage flexibility (case i) was 48% less than without considering it (case ii). As the impact force in the lanyard is the same (because of the absorber), the difference is due to the greater sag (12.24% of the span, in case i, compared to 6.32% in case ii). This is because the flexibility of the anchorage is bigger than that of the cable. In case (i), the cable stretched only 0.16%, while the span contracted 2.40%).

c) The absorber extension Δl_a, considering anchorage flexibility (case i), was 29% less than without considering it (case ii). This is because more energy goes to the elastic deformation, and less is left to the absorber.

B. Limitations of the model and ways of improving it

It will be necessary to make tests in order to validate the model. Many assumptions were made in order to make it easier to solve, even analytically. The testing can show how the method needs to be improved, by modifying some of these assumptions. The numerical procedure can be easily modified, by writing a different potential energy equation and choosing other variables.

1. It was used an static method for calculating a dynamical problem by introducing a fictitious applied force. It was used an equilibrium condition (the principle of minimum total potential energy) in order to calculate a system that is not in equilibrium. The mass has only momentarily stopped before bouncing back, and the different parts of the system might not be in equilibrium.
2. The principle of minimum total potential energy presupposes conservative forces, but parts of the system, as the energy absorbers, are dissipative. These were treated only by the energy balance. Other components, like the lanyard, present energy

dissipation along with elasticity. Should this be taken in consideration, then it might be necessary a different variational principle.

3. The elasticity of the connections wasn't considered. It was assumed a lanyard and a lifeline of a determined length of line with a homogeneous stiffness. The real line is made of many components: line, connections, cable loops, fasteners, each one with a different elasticity, and sometimes difficult to predict way of deforming.
4. It was assumed that the lanyard and the lifeline are perfectly elastic, with constant stiffness. Otherwise, fiber ropes are known to have viscoelastic behavior. Thus, the stiffness will be dependent not only on the elongation, but also, in part, on the elongation speed, and consequently on the falling body speed and on the free fall distance. Beside that, there is energy absorption by the lanyard during the fall arrest. Wire ropes are generally described as featuring a permanent stretch in the first loadings and an approximate elastic behavior after that, but there is not much information as to the dynamic behavior. In order to improve the model, the elastic potential energy of the lanyard and the lifeline $U_{e_i} = \frac{k_i}{2}(\Delta l_i)^2$ could be replaced by the integration of a polynomial adjusted from experimental data of the tension-elongation characteristics of the ropes, as proposed by Baszczyński [20].
5. The lifeline was assumed weightless. The self-weight contributes to increase the tension in the line, by increasing the load, and by changing the shape of the line. In the numerical solution, it's possible to consider the self-weight in a discrete mode, adding some loaded points between the center and the extremes, building a correspondent energy function, and allowing their positions to vary both horizontal and vertically.
6. Only elastic deformation of the anchorage was considered. In real lifelines, sometimes there are some movements during fall arrest that are non linear elastic, due to gaps, rotations, etc.
7. We assumed constant force energy absorbers. Usually, there are ups and downs. An easy improvement is to consider the peak value of the force as the force applied, for the tensions calculation, but use the average value of the force for the absorber length extension calculation.
8. Sensibility to adjustments: it may be difficult that the parameters used in the calculation are properly set in the work site, and kept constant after that. This difficulty is not specific of the present model.
9. Cases that were not treated in the present model:
   a. Falls not in the middle of the span
   b. Multiple spans
   c. Simultaneous or sequential falls
   d. Falls not directly below lifeline
10. Limitations of the optimization software[21]:
   a. A nonlinear problem may have more than one minimum, then, sometimes the software may find a solution that is not the physical one searched. We have to keep this in mind when examining the results. It's a good idea run the Solver starting from several different sets of initial values for the variables.

b. Sometimes, the program stops the search short of the minimum searched. In this case, we run it again, departing from the result found; the energy function will decrease a little bit more, and the result will change a little bit.

c. The optimization software embedded in the spreadsheet used can deal with a maximum of 200 variable cells. If necessary a greater number, it can be used another software, or a program may be written to do the minimization.

V. CONCLUSIONS

A. The model computes the values of the tensions in each component, by considering conjointly the elasticity of the lanyard, the lifeline and the anchorage, and their effects on the energy absorption by each component and on the sag.

B. It was shown that, by considering the elasticity of the anchorage, the maximum tension in the lifeline and on the anchorage can be significantly lower than the calculated without considering it, making it possible to design a lighter structure. It is also possible to design anchorages with an appropriate flexibility, in order to reduce the tensions.

C. When the anchorage deflection is blocked, the results of the proposed method coincide with the results of the usual methods for horizontal lifeline calculation.

D. The energy approach proposed is easier to formulate, making possible the analytical and numerical solution of the three component system.

E. The method developed, using a spreadsheet software and its associated optimization algorithm, for energy function minimization, has several advantages:

1. It uses a readily available software, instead of a specific purpose one.
2. It's possible to write the system parameters, the potential energy function and intermediate formulas directly on the cells, keeping the formulas simple. All the results can also be computed by formulas in the cells.
3. It's easy to try different values for parameters.
4. It's easy to incorporate new features to the model, by changing the energy function, and the variable cells.

F. Testing is necessary to confirm the expected system behavior and to validate the method, and/or to improve it.

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