Lifeline design: calculation of the tensions

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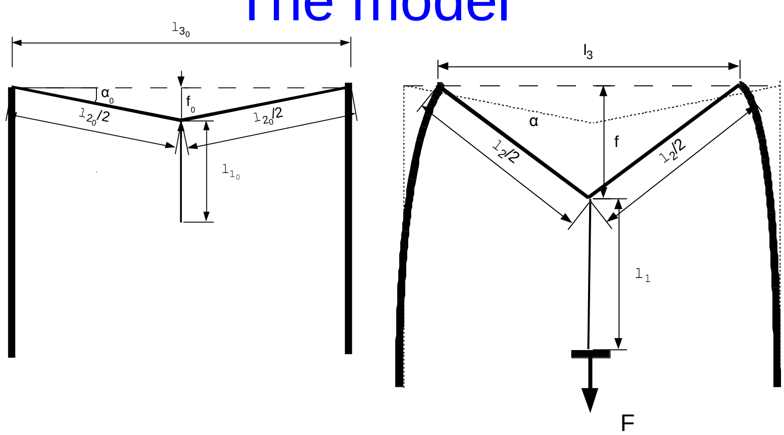
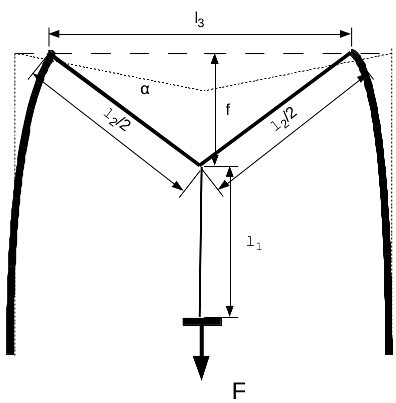
The model



component 1: lanyard
component 2: lifeline
component 3: anchorage

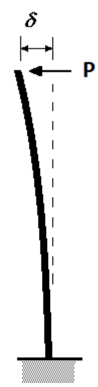
Ministry of **Labour and Employment**

stiffness constants k_i



$$k_1 = \frac{k_r}{l_{1_0}}$$

$$k_2 = \frac{EA_m}{l_{2_0}}$$



$$k_3 = \frac{P}{2\delta}$$

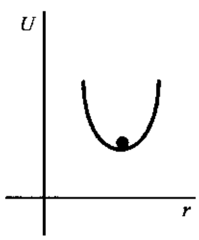
parameters, variables and expressions

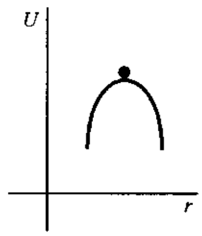
- Parameters (constants):
  - the elastic constants k_1, k_2, k_3
  - initial lengths $l_{1_0}, l_{2_0}, l_{3_0}$
  - applied force F
- Variables:
  - length variations $\Delta l_1, \Delta l_2, \Delta l_3$
- Expressions (results)
  - final lengths $l_1 = l_{1_0} + \Delta l_1$

    $l_2 = l_{2_0} + \Delta l_2$

    $l_3 = l_{3_0} - \Delta l_3$
  - final tensions $T_i = k_i \Delta l_i$

Principle of minimum total potential energy





$$\frac{dU(r)}{dr} = 0.$$

- *Among all the possible displacements consistent with the reactions, the correct state of displacement is that which minimizes the total potential energy.*

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energy expressions

- elastic potential energy

$$U_e = \frac{k_1}{2}(\Delta l_1)^2 + \frac{k_2}{2}(\Delta l_2)^2 + \frac{k_3}{2}(\Delta l_3)^2$$

- potential energy function of the external loads $U_F = -F(f + l_1)$
  - sag $f = \frac{1}{2}\sqrt{l_2^2 - l_3^2}$
- total potential energy $U = U_e + U_F$

Principle of Energy conservation

$$\Delta E = 0 \Rightarrow \begin{cases} \Delta U_g + \Delta U_e = 0 & \leftarrow \text{without absorber} \\ \Delta U_g + \Delta U_e + W_a = 0 & \leftarrow \text{with absorber} \end{cases}$$

$$\Delta U_g = mg\Delta h = mg\left[-\left(f + l_1\right) - \left(y_0\right)\right] = -mg\left(y_0 + f + l_1\right)$$

$$\Delta U_e = U_e - U_{e_0} \qquad U_e = \frac{k_1}{2}(\Delta l_1)^2 + \frac{k_2}{2}(\Delta l_2)^2 + \frac{k_3}{2}(\Delta l_3)^2$$

$$U_{e_0} = \begin{cases} 0 & \leftarrow l_{2_0} \geq l_{3_0} \\ \dfrac{\left(l_{3_0} - l_{2_0}\right)^2}{2\left(\frac{1}{k_2} + \frac{1}{k_3}\right)} & \leftarrow l_{2_0} < l_{3_0} \end{cases}$$

$$W_a = \Delta l_a T_a$$

Analytical solution

$$\frac{\partial U}{\partial l_1} = 0 \Rightarrow k_1(l_1 - l_{1_0}) - F = 0$$

$$\frac{\partial U}{\partial l_2} = 0 \Rightarrow k_2(l_2 - l_{2_0}) - \frac{F}{2}\frac{l_2}{\sqrt{l_2^2 - l_3^2}} = 0$$

$$\frac{\partial U}{\partial l_3} = 0 \Rightarrow k_3(l_{3_0} - l_3) - \frac{F}{2}\frac{l_3}{\sqrt{l_2^2 - l_3^2}} = 0$$

Angle equations

$$tan(\alpha) - \frac{sin(\alpha)}{l_{3_0} / l_{2_0}} = \left(\frac{1}{k_2} + \frac{1}{k_3}\right)\frac{F}{2l_{3_0}}$$

$$k_3 tan(\alpha) + k_2 \frac{sin(\alpha)}{l_{3_0} / l_{2_0}} = (k_2 + k_3)\frac{2f}{l_{3_0}}$$

Energy absorber: 4 cases

- energy absorber in the lanyard
- in the lifeline
- none
- both

example input data

- lanyard: nylon $\varnothing = 12.7mm(1/2")$ $l_{1_0} = 1.20m$ $k_1 = 30000\,N/m$
- lifeline: wire rope 6x19 IWRC; $\varnothing = 12.7mm(1/2")$ $l_{2_0} = 6.23m$ $k_2 = 1200000\,N/m$
- anchorage: steel round tube 101.6 mm (4“); thickness t = 5.74 mm; $k_3 = 80000\,N/m$ $l_{3_0} = 6.20m$
- lanyard EA: force T_a = 6000 N
- lifeline EA: force T_a =10000 N

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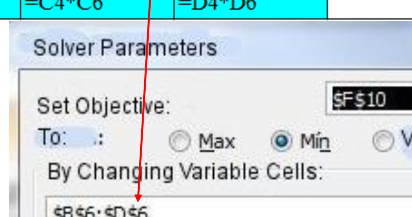
	A	B	C	D	E	F
1		lanyard	lifeline	span	sag	sum
2	ref.	1	2	3	f	
3	F	6000				
4	k_i	30000	1200000	80000		
5	l_{i0}	1.2	6.23	6.2	=0.5*(C5^2-D5^2)^0.5	
6	Δl	0	0	0		
7	l_i	=B5+B6	=C5+C6	=D5-D6	=0.5*(C7^2-D7^2)^0.5	
8	U_e	=B4/2*B6^2	=C4/2*C6^2	=D4/2*D6^2		=SOMA(B8:E8)
9	U_f	=-B7*$B3			=-E7*$B3	=SOMA(B9;E9)
10	U					=SOMA(F8:F9)
11	T_i	=B4*B6	=C4*C6	=D4*D6		



Absorber in lanyard - Results

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	A	B	C	D	E	F	G
1		lanyard	lifeline	span	sag	sum	units
2	ref.	1	2	3	f		
3	F	6000					N
4	k_i	30000	1200000	80000			N/m
5	l_{i0}	1.2000	6.2300	6.2000	0.3053		m
6	Δl	0.2000	0.0102	0.1488			m
7	l_i	1.4000	6.2402	6.0512	0.7622		m
8	U_e	600	63	886		1549	J
9	U_f	-8400			-4573	-12973	J
10	U					-11424	J
11	T_i	6000	12280	11908			N

Results – Lanyard absorber

cases	Δl1	Δl2	Δl3	l1	l2	l3	f	f%	e2%	e3%
V: 1, 2, 3	0.20	0.010	0.15	1.40	6.24	6.05	0.76	12.24%	0.16%	2.40%
V: 1, 2 F: 3	0.20	0.020		1.40	6.25	6.20	0.39	6.32%	0.32%	
V: 1 F: 2, 3	0.20			1.40	6.23	6.20	0.31	4.90%		

Cases	T1	T2	T3	Δla	Ue1	Ue2	Ue3	Ue	Wa	Ug
V: 1, 2, 3	6000	12280	11908	0.17	600	63	886	1549	861	2410
V: 1, 2 F: 3	6000	23803		0.24	600	236		836	1213	2049
V: 1 F: 2, 3	6000			0.27	600			600	1362	1962

Cases	Ue1	Ue2	Ue3	Ue	Wa	Ug
V: 1, 2, 3	25%	3%	37%	64%	36%	100%
V: 1, 2 F: 3	29%	12%		41%	59%	100%
V: 1 F: 2, 3	31%			31%	69%	100%

No absorber - Results

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	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	
1					lanyard	lifeline	span	sag		mass	gravity	initial position	free fall distance	fall factor		inpu
2				ref.	1	2	3	f_0 (m)		m (kg)	g (m/s²)	y_0 (m)	$h = y_0 + l_{10} + f_0$	r		vari
3				k_i (N/m)	30000	1,20E+06	8,00E+04			100	9,81	0,29467	1,8	1,5		resu
4				l_{i0} (m)	1,2	6,23	6,2	0,3053								min
5		lanyard	lifeline	span												
6	F (N)	Δl_1 (m)	Δl_2 (m)	Δl_3 (m)	l_1 (m)	l_2 (m)	l_3 (m)	f (m)	U_e (J)	U_f (J)	U (J)	U_g (J)	T_1 (N)	T_2 (N)	T_3 (N)	
7	7500,0	0,2500	0,0120	0,1737	1,45	6,24	6,03	0,81	2230,19	-5684,73	-3454,54	2509,36	7500,0	14390,5	13893,4	
8	7600,0	0,2533	0,0121	0,1753	1,45	6,24	6,02	0,82	2279,18	-5809,84	-3530,66	2515,73	7600,0	14526,3	14020,5	
9	7700,0	0,2567	0,0122	0,1768	1,46	6,24	6,02	0,82	2328,61	-5936,03	-3607,43	2522,07	7700,0	14661,6	14147,1	
10	7800,0	0,2600	0,0123	0,1784	1,46	6,24	6,02	0,82	2378,47	-6063,31	-3684,84	2528,38	7800,0	14796,3	14273,1	
11	7900,0	0,2633	0,0124	0,1800	1,46	6,24	6,02	0,83	2428,77	-6191,67	-3762,90	2534,66	7900,0	14930,4	14398,5	
12	8000,0	0,2667	0,0126	0,1815	1,47	6,24	6,02	0,83	2479,51	-6321,10	-3841,59	2540,93	8000,0	15064,1	14523,3	
13	8100,0	0,2700	0,0127	0,1831	1,47	6,24	6,02	0,83	2530,69	-6451,61	-3920,92	2547,16	8100,0	15197,2	14647,6	
14	8200,0	0,2733	0,0128	0,1846	1,47	6,24	6,02	0,83	2582,30	-6583,18	-4000,89	2553,37	8200,0	15329,8	14771,4	
15	8300,0	0,2767	0,0129	0,1862	1,48	6,24	6,01	0,84	2634,34	-6715,83	-4081,49	2559,56	8300,0	15462,0	14894,6	
16	8400,0	0,2800	0,0130	0,1877	1,48	6,24	6,01	0,84	2686,82	-6849,53	-4162,72	2565,73	8400,0	15593,6	15017,3	
17	8500,0	0,2833	0,0131	0,1892	1,48	6,24	6,01	0,84	2739,72	-6984,29	-4244,57	2571,87	8500,0	15724,7	15139,5	
18	8600,0	0,2867	0,0132	0,1908	1,49	6,24	6,01	0,85	2793,06	-7120,11	-4327,05	2577,99	8600,0	15855,4	15261,2	
19	8700,0	0,2900	0,0133	0,1923	1,49	6,24	6,01	0,85	2846,83	-7256,99	-4410,15	2584,09	8700,0	15985,6	15382,4	
20	8800,0	0,2933	0,0134	0,1938	1,49	6,24	6,01	0,85	2901,03	-7394,91	-4493,88	2590,16	8800,0	16115,4	15503,1	
21	8900,0	0,2967	0,0135	0,1953	1,50	6,24	6,00	0,86	2955,66	-7533,88	-4578,22	2596,22	8900,0	16244,7	15623,3	
22											-60101,8					
23																

Results – No absorber

cases	F (N)	Δl1	Δl2	Δl3	l1	l2	l3	f	f%
V:1,2,3	8136.4	0.27	0.013	0.18	1.47	6.24	6.02	0.83	13.4%
V:1,2 F:3	10016.8	0.33	0.030		1.53	6.26	6.20	0.43	7.0%
V:1 F:2,3	11320.7	0.38			1.58	6.23	6.20	0.31	4.9%

cases	e2%	e3%	T1	T2	T3	Ue1	Ue2	Ue3	Ue
V:1,2,3	0.20%	3.0%	8136	15246	14693	1103	97	1349	2549
V:1,2 F:3	0.48%		10017	36209		1672	546		2219
V:1 F:2,3			11321			2136			2136

cases	Uf	U	Ug	Ue1	Ue2	Ue3	Ue	Ug
V:1,2,3	-6499	-3950	2549	43%	4%	53%	100%	100%
V:1,2 F:3	-4623	-2404	2219	75%	25%	0%	100%	100%
V:1 F:2,3	-4272	-2136	2136	100%	0%	0%	100%	100%

Absorber in the line - Results

	A	B	C	D	E	F	G	H	I	J
1		lanyard	lifeline	span	sag	sum	units			
2	ref.	1	2	3	f			input data	value	
3	T_a		10000				N	variables	value	
4	k_i	30000	1200000	80000			N/m	**equal 0**	formula	
5	l_{i0}	1.2000	6.2300	6.2000	0.3053		m	results	formula	
6	Δl	0.2009	0.0083	0.1192			m			
7	l_i	1.4009	6.3774	6.0808	0.9611		m	initial sag f_0 %	4.9%	
8	U_e	606	42	568		1216	J	final sag f %	15.4%	
9	U_f	0			0	0	J	cable elongation e_2 %	0.13%	
10	U					1216	N	span shortening e_3 %	1.92%	
11	T_i	6028	10000	9535						
12										
13	mass	gravity	initial position	free fall distance	fall factor	grav. energy	absorp tion	absorber extension	absorbe r exten-	
14	m (kg)	g (m/s²)	y_0 (m)	$h= y_0 + l_{10} + f_0$	$r = h / l_{10}$	Ug (J)	U_g-U_e	$(U_g-U_e)/T_a-\Delta l_a$	Δl_a (m)	$\Delta l_{a,max}$
15	100	9.81	0.2947	1.8000	1.5	2606	1391	0.000000	0.1391	0.60

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Absorber in both line and lanyard - Results

	A	B	C	D	E	F	G	H	I	J	K	L
1		lanyard	lifeline	span	sag	sum	units			cos α_0	0.995	
2	ref.	1	2	3	f			input data	value	α_0	0.098	rad
3	T_a	6000	10000				N	variables	value	sen α_2	0.300	
4	k_i	30000	1200000	80000			N/m	to minimize	formula	α_2	0.305	rad
5	l_{i0}	1.2000	6.2300	6.2000	0.3053		m	results	formula	cos α_2	0.954	
6	Δl	0.2000	0.0083	0.1192			m			$l_{2'}$	6.37	m
7	l_i	1.4000	6.2383	6.0808	0.6966		m	initial sag f_0 %	4.9%	Δl_{ll}	0.14	m
8	U_e	600	42	569		1210	J	final sag f %	11.2%	$\Delta l_{ll,max}$	0.60	m
9	U_f	-8400			-4180	-12580	J	cable elongation e_2 %	0.13%	W_{ll}	1360	J
10	U					-11369	J	span shortening e_3 %	1.92%			
11	T_i	6000	10000	9539			N					
12												
13	mass	gravity	initial position	free fall distance	fall factor	grav. energy	absorp tion	absorber extension				
14	m (kg)	g (m/s²)	y_0 (m)	$h= y_0 + l_{10} + f_0$	$r = h / l_{10}$	Ug (J)	$W_{ly}=U_g-U_e-$	Δl_{ly} (m)	$\Delta l_{a, max}$ (m)			
15	100	9.81	0.2947	1.8000	1.5	2346	-225	-0.0448	0.45			

Some limitations and possible improvements

- Testings are needed to validate the model
- Lanyard and lifeline assumed elastic
- Lifeline assumed weightless
- Absorber force assumed constant
- Incorporate the post structures to the method

Conclusions

1. The model proposed:
   a. Model considers conjointly the elasticity of the lanyard, the lifeline and the anchorage, and their effects on the energy absorption by each component and on the sag.
   b. Maximum tension in the lifeline and on the anchorage can be significantly lower than without considering the anchorage flexibility
   c. When the anchorage deflection is blocked, results coincide with usual methods.
   d. Testing is necessary to validate the model

Conclusions

2. The energy approach:
   a. is easier to formulate, making possible the analytical and numerical solution of the three component system.
3. The numerical method:
   a. uses readily available software
   b. keeps formulas simple
   c. easy to try different values for parameters
   d. easy to incorporate new features
   e. can be applied to other small structures

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Questions?

Thank you very much!

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